Alternative Direction Finding Methods for Oceanographic HF Radars

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ROMS Ocean Model

40 km
How can we resolve complex flow structures?
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1) Revisit Signal Processing
How can we resolve complex flow structures?

1) Revisit Signal Processing
2) Improve Understanding of Errors
How can we resolve complex flow structures?

1) Revisit Signal Processing
2) Improve Understanding of Errors
   – Quantify them
1) Revisit Signal Processing
Alternative Direction Finding Methods

MUSIC
W-MUSIC – Weights on eigenvectors
MLE-AP – Maximum Likelihood
SML – Stochastic ML
WSF – Weighted Subspace Fitting
Simulation-Based Stress Test

- 2 signal sources
- 0.2 beamwidth separation
- 10 ‘snapshots’
How can we resolve complex flow structures?

1) Revisit Direction Finding
2) Improve Understanding of Errors
   – Quantify them
2) Improve Understanding of Errors – Quantify them
MUSIC, Maximum Likelihood, and Cramer–Rao Bound

PETRE STOICA AND ARYE NEHORAI, MEMBER, IEEE
MUSIC, Maximum Likelihood, and Cramer–Rao Bound

PETRE STOICA AND ARYE NEHORAI, MEMBER, IEEE

\[ E(\hat{\theta}_i - \theta_i)^2 = \]

**MUSIC Error Variance**
MUSIC, Maximum Likelihood, and Cramer–Rao Bound

PETRE STOICA AND ARYE NEHORAI, MEMBER, IEEE

\[ E(\hat{\theta}_i - \theta_i)^2 = \frac{\sigma}{2K} \]

- Noise Level
- MUSIC Error Variance
- Snapshots
MUSIC, Maximum Likelihood, and Cramer–Rao Bound

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\[
E(\hat{\theta}_i - \theta_i)^2 = \frac{\sigma}{2K} \frac{dA^H}{d\theta} E_n E_n^H \frac{dA}{d\theta}
\]
MUSIC, Maximum Likelihood, and Cramer–Rao Bound

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\[ E(\hat{\theta}_i - \theta_i)^2 = \frac{\sigma}{2K} \frac{dA^H}{d\theta} E_n E_n^H \frac{dA}{d\theta} \]

Noise Level

MUSIC Error Variance

Snapshots

Antenna Pattern
MUSIC, Maximum Likelihood, and Cramer–Rao Bound

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\[ E(\hat{\theta}_i - \theta_i)^2 = \frac{\sigma}{2K} \cdot \frac{A^H}{\sum_{k=1}^{N} \frac{\lambda_k}{(\sigma - \lambda_k)^2} S_k S_k^H} A \]

- Noise Level
- Signal Eigenvalues
- MUSIC Error Variance
- Snapshots
- Antenna Pattern
SUMMARY

1) MLE has slight advantage
   - Improved coverage?
   - 5-10x computations
   - First Order and Detection are important

2) MUSIC errors can be estimated
   - First Order, Detection and APM errors not accounted for
“the choice of the appropriate algorithm will depend on the … particular application"

–Van Trees 2002
The “particular application"
Low K
Low N
N near M
Low SNR
Uncorrelated for > 1 deg (Barrick and Snyder 1972)
APM
compare with Ziskind and Wax, 1988, figure 2
Alternative DOA Methods

Given:
\[ a(\theta) = [\cos(\theta) \cos(\theta - 90) \ldots 1]^T \]
\[ A(\theta) = [a(\theta)_i \ldots a(\theta)_D] \]

**MUSIC**
\[ P_{MUSIC} = \frac{1}{a^H(\theta)E_N E_N^H a(\theta)} \]

**W-MUSIC** (weighted MUSIC - Min Norm, and ..)
\[ P_{W-MUSIC} = \frac{1}{a^H(\theta)(E_N E_N^H W(E_N E_N^H)) a(\theta)} \]

**MLE (AP)** (Kind of ML is this?)
\[ P_{MLE} = \text{trace}(A(A^H A)^{-1} A^H R) \]

**WSF**
“... either CML-AP or MODE-AP provided the best threshold behavior” -VT02

"MLE outlier production … occurs at a significantly lower SNR than for MUSIC" -TF09

"... some of these perform almost as well as the ML algorithms" -VT02

"Thus, the Root-WSF algorithm is a strong candidate for the “best” method for ULAs." KV1996

" the new estimator is expected to perform better than other MUSIC-type estimators" SN89
Oke et al. (2002):
“further efforts to understand error structure in HF radar derived data are clearly warranted”

O’Donnell et al. (2005) recommends:
“develop … uncertainty estimates so that search areas can be modeled more effectively”

\[
\begin{bmatrix}
X_1 \\
X_2 \\
\vdots \\
X_M
\end{bmatrix} =
\begin{bmatrix}
a(\theta_1) & a(\theta_2) & \cdots & a(\theta_D)
\end{bmatrix}
\begin{bmatrix}
F_1 \\
F_2 \\
\vdots \\
F_D
\end{bmatrix} +
\begin{bmatrix}
W_1 \\
W_2 \\
\vdots \\
W_M
\end{bmatrix}
\]

or

\[
X = AF + W.
\]

\[
S = APA^* + \lambda_{\text{min}} S_0,
\]

\[
P_{MU}(\theta) = \frac{1}{a^*(\theta)E_N E_N^* a(\theta)}.
\]
MUSIC

- Signal and noise subspaces
- More elements than emitters
- Noise Gaussian
MUSIC

- Signal and noise subspaces
- More elements than emitters
- Noise Gaussian
Doppler pre-processing

![Doppler pre-processing graph](image_url)
The graph shows the relationship between radial velocity (cm s⁻¹) and bearing (° cwN). The data points are represented by different markers and colors:

- **Vr input** (solid blue line)
- **MUSIC (RMSD = 2.0027)** (green asterisks)
- **MLE-AP (RMSD = 1.981)** (red circles)

The radial velocity decreases as the bearing increases from -80° to 80°, with the MUSIC and MLE-AP models showing a higher degree of fit compared to the input data.
MUtiple Signal Classification
“MUSIC”
$C = \begin{bmatrix}
C_{11} & C_{12} & C_{13} \\
C_{21} & C_{22} & C_{23} \\
C_{31} & C_{32} & C_{33}
\end{bmatrix} = \begin{bmatrix}
0.2162 & 0.0303 - 0.0090i & 0.3170 - 0.0063i \\
0.0303 + 0.0090i & 0.0436 & -0.0091 + 0.0213i \\
0.3170 + 0.0063i & -0.0091 - 0.0213i & 0.5416
\end{bmatrix}$
$\mathbf{C} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} 0.2162 & 0.0303 - 0.0090i & 0.3170 - 0.0063i \\ 0.0303 + 0.0090i & 0.0436 & -0.0091 + 0.0213i \\ 0.3170 + 0.0063i & -0.0091 - 0.0213i & 0.5416 \end{bmatrix}$
MUSIC, Maximum Likelihood, and Cramer–Rao Bound

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\[ E(\hat{\theta}_i - \theta_i)^2 = \frac{\sigma}{2K} \left[ A^* \sum_{k=1}^{N} \frac{\lambda_k}{(\sigma - \lambda_k)^2 s_k s_k^*} \right] A \]

\[ \frac{\partial A^*}{\partial \theta} E_N E_N^H \frac{\partial A}{\partial \theta} \]
MUSIC, Maximum Likelihood, and Cramer–Rao Bound

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\[ E(\hat{\theta}_i - \theta_i)^2 = \frac{\sigma}{2K} \left[ A^* \left( \sum_{k=1}^{N} \frac{\lambda_k}{(\sigma - \lambda_k)^2} s_k s_k^* \right) A \right] \]

MUSIC Error Variance
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\[ E(\hat{\theta}_i - \theta_i)^2 = \frac{\sigma}{2K} \frac{A^* \left[ \sum_{k=1}^{N} \frac{\lambda_k}{(\sigma - \lambda_k)^2} s_k s_k^* \right] A}{\frac{dA^*}{d\theta} E_N E_N^H \frac{dA}{d\theta}} \]
MUSIC, Maximum Likelihood, and Cramer–Rao Bound

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\[
E(\hat{\theta}_i - \theta_i)^2 = \frac{\sigma}{2K} A^* \left[ \sum_{k=1}^{N} \frac{\lambda_k}{(\sigma - \lambda_k)^2} s_k^* s_k \right] A
\]

\[
\frac{dA^*}{d\theta} E_N E_N^H \frac{dA}{d\theta}
\]

MUSIC Error Variance

Antenna Pattern

Snapshots
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\]